# Courbes de niveau

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La première version de ce document est parue en janvier 2006. Ce document a servi à la création de figures pour illustrer un document pdf mis à la disposition de mes élèves du cours Calcul intégral avancé (cours EEE). Ce document pdf est disponible sur mon site Interent. Je rend donc disponible le fichier source Maple sans aucune documentation pédagogique.

Bonne lecture à tous !

* Ce document Maple est exécutable avec la version 2020.2

## Initialisation

```
> restart;
```

```
> with(plots,contourplot,contourplot3d,display,setoptions,setoptions3d,
  spacecurve);
  with(plottools,transform);
```

$$[contourplot, contourplot3d, display, setoptions, setoptions3d, spacecurve]$$

$$[transform] \quad \textbf{(1.1)}$$

```
> setoptions(size=[300,300],labels=[x,y],tickmarks=[12,12],axesfont=
  [TIMES,ROMAN,8],labelfont=[TIMES,ROMAN,8]);
  setoptions3d(size=[300,300],labels=[x,y,z],tickmarks=[4,4,4],
  axesfont=[TIMES,ROMAN,8],labelfont=[TIMES,ROMAN,8]);
```

L'initialisation suivante permettra d'avoir plus de lisibilité des nombres décimaux en supprimamt les zéros non significatifs à la fin d'un nombre.

```
> interface(typesetting=extended); # Pour s'assurer le niveau de
  composition étendue
  Typesetting:-Settings(striptrailing=true);
```

$$extended$$

$$false \quad \textbf{(1.2)}$$

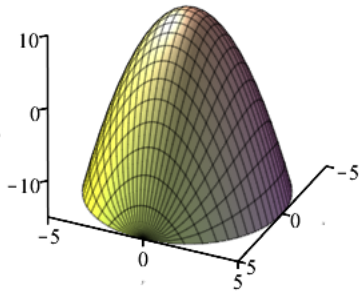
## Paraboloïde circulaire

```
> f:=(x,y)->10-x^2-y^2:
  z=f(x,y);
```

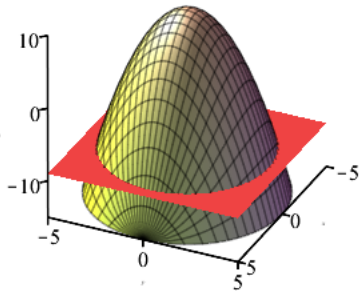
$$z = -x^2 - y^2 + 10 \quad \textbf{(2.1)}$$

```
> Surface:=plot3d([x,y,f(x,y)],x=-5..5,y=-sqrt(25-x^2)..sqrt(25-x^2)):
> display(Surface,axes=framed,
                  orientation=[25,60],
                  lightmodel=light3,
```

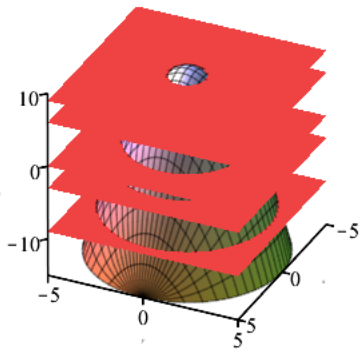
```
                    shading=XY);
```



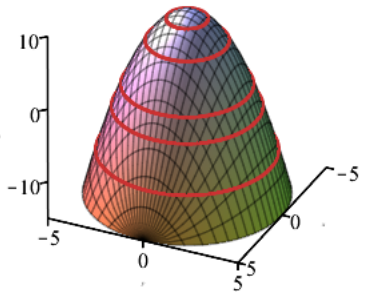

```
> Plan1:=plot3d([x,y,-9],x=-5..5,y=-5..5,
           grid=[20,20],
           style=patchnogrid,
           color=orange):
> display([Surface,Plan1],axes=framed,orientation=[25,60],shading=XY);
```



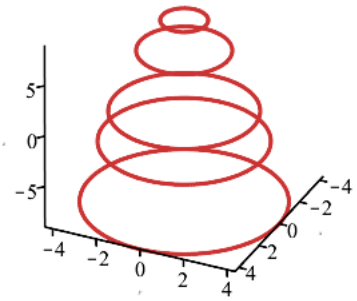

```
> Opts:=grid=[20,20],style=patchnogrid,color=orange:
> Plan2:=plot3d([x,y,-3],x=-5..5,y=-5..5,Opts):
  Plan3:=plot3d([x,y,0],x=-5..5,y=-5..5,Opts):
  Plan4:=plot3d([x,y,6],x=-5..5,y=-5..5,Opts):
  Plan5:=plot3d([x,y,9],x=-5..5,y=-5..5,Opts):
> display([Surface,Plan||(1..5)],
           axes=framed,
           orientation=[25,60]);
```



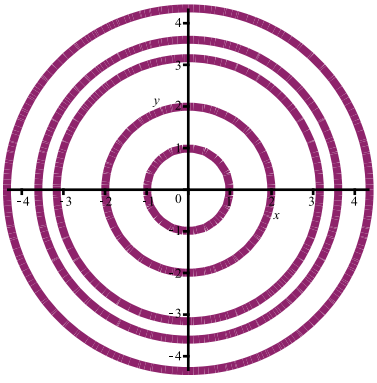
```
> Opts:=thickness=3,color=orange:
> Trace1:=spacecurve([sqrt(19)*cos(t),sqrt(19)*sin(t),-9],t=0..2*Pi,
  Opts):
  Trace2:=spacecurve([sqrt(13)*cos(t),sqrt(13)*sin(t),-3],t=0..2*Pi,
  Opts):
  Trace3:=spacecurve([sqrt(10)*cos(t),sqrt(10)*sin(t),0],t=0..2*Pi,
  Opts):
  Trace4:=spacecurve([sqrt(4)*cos(t),sqrt(4)*sin(t),6],t=0..2*Pi,Opts):
  Trace5:=spacecurve([sqrt(1)*cos(t),sqrt(1)*sin(t),9],t=0..2*Pi,Opts):
> display([Surface,Trace||(1..5)],axes=framed,orientation=[25,60]);
```



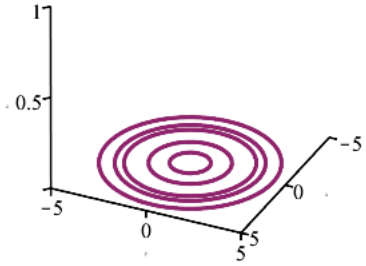

```
> display([Trace||(1..5)],axes=framed,orientation=[25,60]);
```



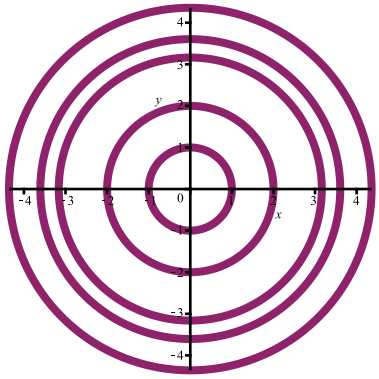

```
> Courbes_de_niveau:=contourplot(f(x,y),x=-5..5,y=-5..5,contours=[-9,
  -3,0,6,9],grid=[40,40], color=maroon,thickness=3):
> display(Courbes_de_niveau,scaling=constrained);
```



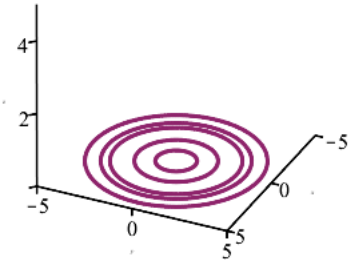
```
> t:=transform((x,y)->[x,y,0]):
> display(t(Courbes_de_niveau),axes=framed,orientation=[25,60],
          view=[-5..5,-5..5,0..1]);
```



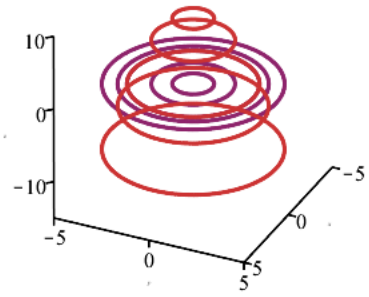

```
> Courbes_de_niveau_bis:=seq((plot([k*sin(t),k*cos(t),t=0..2*Pi],
  thickness=3,color=maroon),k=[sqrt(19),sqrt(13),sqrt(10),sqrt(4),sqrt
  (1)])):
> display(Courbes_de_niveau_bis,scaling=constrained);
```




```
> display(seq(t(Courbes_de_niveau_bis[k]),k=1..5),axes=framed,
          orientation=[25,60],
          view=[-5..5,-5..5,0...5]);
```




```
> display([seq(t(Courbes_de_niveau_bis[k]),k=1..5),Trace||(1..5)],
           axes=framed,orientation=[25,60],
           view=[-5..5,-5..5,-15...10]);
```

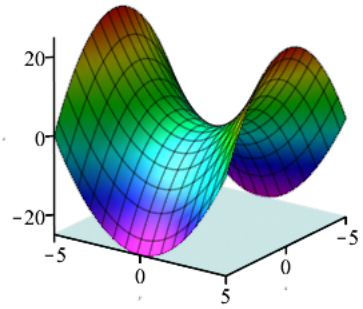



# Paraboloïde hyperbolique

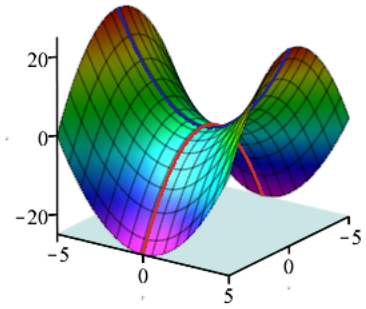
```
> f:=(x,y)->y^2-x^2:
  z=f(x,y);
```

$$z = -x^2 + y^2$$

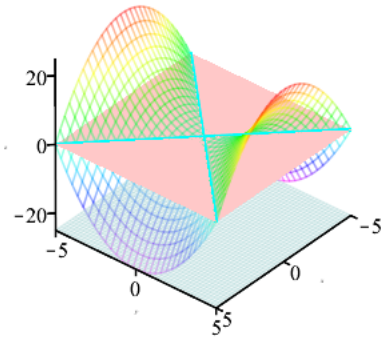
```
> macro(domaine=COLOR(RGB, .6863, .8471, .8471)):
  macro(plan=COLOR(RGB, 1.0000, .6667, .6667)):
> Domaine:=plot3d([x,y,f(5,0)],x=-5..5,y=-5..5,
          style=wireframe,
          color=domaine):
  Surface:=plot3d([x,y,f(x,y)],x=-5..5,y=-5..5,
           grid=[30,30]):
> display([Surface,Domaine],
          axes=framed,
          shading=zhue,
          orientation=[35,70]);
```




```
> A:=spacecurve([0,y,y^2,y=-5..5],thickness=3,color=navy):
  B:=spacecurve([x,0,-x^2+.05,x=-5..5],thickness=3,color=orange):
> display([Surface,Domaine,A,B],
          axes=framed,
          shading=zhue,
          orientation=[35,70]);
```

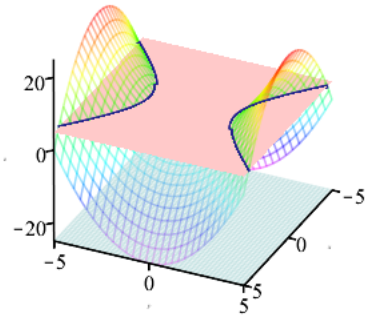



```
> Surface:=plot3d([x,y,f(x,y)],x=-5..5,y=-5..5,
            grid=[30,30],
            style=wireframe):
> Plan1:=plot3d([x,y,0],x=-5..5,y=-5..5,
           grid=[40,40],
           style=patchnogrid,
           color=plan):
> Inter1:=spacecurve([x,x,0,x=-5..5],thickness=3,color=cyan):
  Inter2:=spacecurve([x,-x,0,x=-5..5],thickness=3,color=cyan):
> display(Surface,Plan1,Domaine,Inter1,Inter2,
          axes=framed,
          shading=zhue,
          orientation=[40,55]);
```

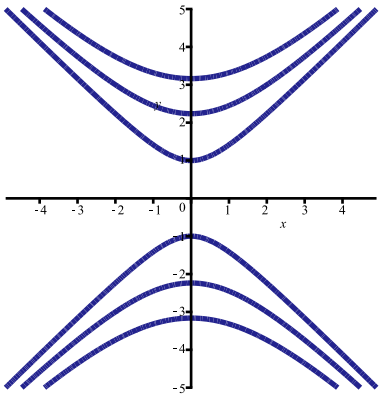



```
> Plan2:=plot3d([x,y,5],x=-5..5,y=-5..5,
           numpoints=400,
           style=patchnogrid,
           color=plan):
> Intersection1:=spacecurve([x,sqrt(x^2+5),5,x=-4.5..4.5],thickness=3,
  color=navy,numpoints=600):
  Intersection2:=spacecurve([x,-sqrt(x^2+5),5,x=-4.5..4.5],thickness=3,
  color=navy,numpoints=600):
> display(Surface,Domaine,Plan2,Intersection1,Intersection2,
          axes=framed,
          shading=zhue,
```

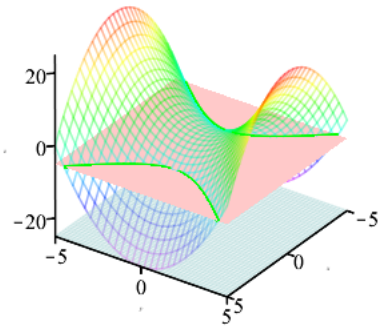
```
          orientation=[25,60]);
```



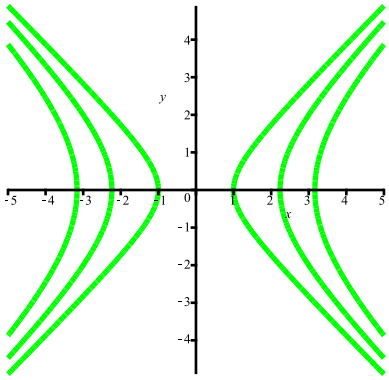

```
> Courbes1:=contourplot(f(x,y),x=-5..5,y=-5..5,grid=[60,60],contours=
  [1,5,10],color=navy,thickness=2):
> display(Courbes1,scaling=constrained);
```



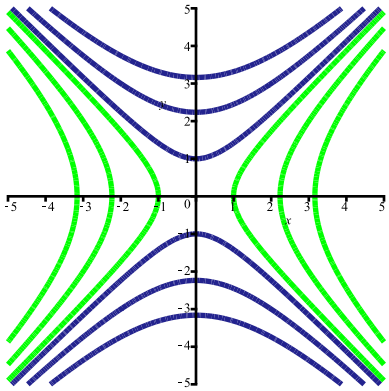

```
> Plan3:=plot3d([x,y,-5],x=-5..5,y=-5..5,
          numpoints=400,
          style=patchnogrid,
          color=plan):
> Intersection1:=spacecurve([x,sqrt(x^2-5),-5,x=-5..5],thickness=3,
  color=green,numpoints=600):
  Intersection2:=spacecurve([x,-sqrt(x^2-5),-5,x=-5..5],thickness=3,
  color=green,numpoints=600):
> display(Surface,Domaine,Plan3,Intersection1,Intersection2,
        axes=framed,
        shading=zhue,
        orientation=[35,60]);
```



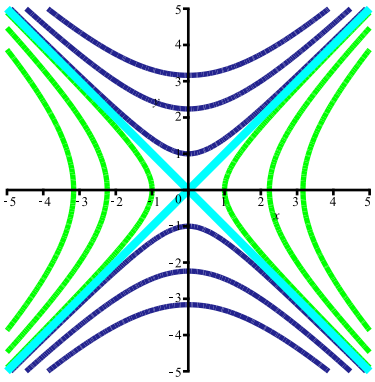

```
> Courbes2:=contourplot(f(x,y),x=-5..5,y=-5..5,grid=[60,60],contours=
  [-10,-5,-1],color=green,thickness=2):
> display(Courbes2,scaling=constrained);
```



```
> display(Courbes1,Courbes2,scaling=constrained);
```



```
> t := transform((x,y,z) -> [x,y]):
> display(Courbes1,Courbes2,t(Inter1),t(Inter2),scaling=constrained);
```

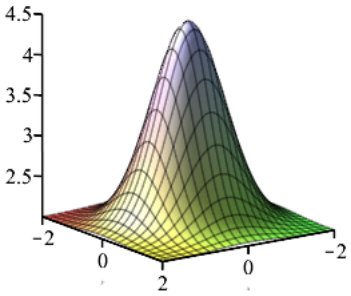


# Surface en forme de cloche

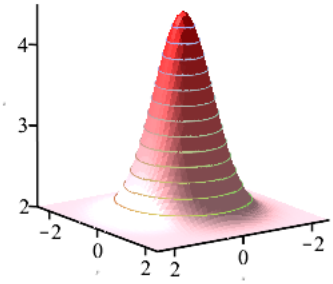
```
> f:=(x,y)->2.5*exp(-x^2-y^2)+2:
  z=f(x,y);
```

$$z = 2.5\,e^{-x^2-y^2} + 2 \qquad (4.1)$$

```
> plot3d([x,y,f(x,y)],x=-2..2,y=-2..2,axes=framed,grid=[45,45]);
```

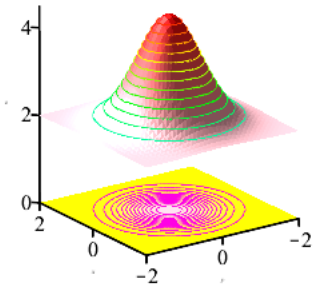



```
> Surface_1:=contourplot3d([x,y,f(x,y)],x=-2.5..2.5,y=-2.5..2.5,
                numpoints=1800,contours=13,
                filled=true,coloring=[pink,red]):
> display(Surface_1,axes=framed);
```




```
> Courbes_1:=contourplot(f(x,y),x=-2..2,y=-2..2,contours=13,
              numpoints=2400,
```

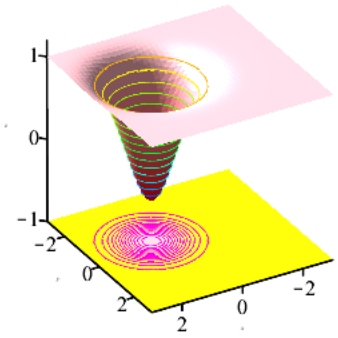
```
              filled=true,coloring=[yellow,pink]):
> t:=transform((x,y)->[x,y,0]):
> display([Surface_1,t(Courbes_1)],
           shading=ZHUE,
           style=patchcontour,
           axes=framed,
           view=[-2..2,-2..2,0..4.5],
           orientation=[145,70],
           font=[TIMES,ROMAN,12],
           scaling=constrained);
```



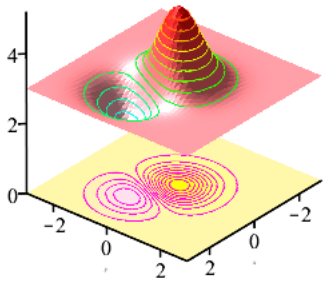

```
> g:=(x,y)->-2.5*.6*exp(-(x-1)^2-(y+.5)^2)+1:
  z=g(x,y);
```

$$z = -1.5\,e^{-(x-1)^2-(y+0.5)^2} + 1$$

```
> Surface_2:=contourplot3d([x,y,g(x,y)],x=-3..3,y=-3..3,
              numpoints=1800,contours=13,
              filled=true,coloring=[red,pink]):
> Courbes_2:=contourplot(g(x,y),x=-3..3,y=-3..3,
              numpoints=2400,filled=true,contours=13,
              coloring=[pink,yellow]):
> t:=transform((x,y)->[x,y,-1]):
> display([Surface_2,t(Courbes_2)],
            shading=ZHUE,axes=framed,
            view=[-3..3,-3..3,-1..1.2],
            orientation=[60,60],font=[TIMES,ROMAN,12]);
```



```
> Surface_3:=contourplot3d([x,y,f(x,y)+g(x,y)],x=-3..3,y=-3..3,
                numpoints=1800,
                contours=13,
                filled=true,
                coloring=[pink,red]):
> Courbes_3:=contourplot(f(x,y)+g(x,y),x=-3..3,y=-3..3,
                numpoints=2400,
                filled=true,contours=13,
                coloring=[pink,yellow]):
> t:=transform((x,y)->[x,y,0]):
> display([Surface_3,t(Courbes_3)],
            shading=ZHUE,axes=framed,
            view=[-3..3,-3..3,0..5.2],
            orientation=[40,60],
            font=[TIMES,ROMAN,12]);
```



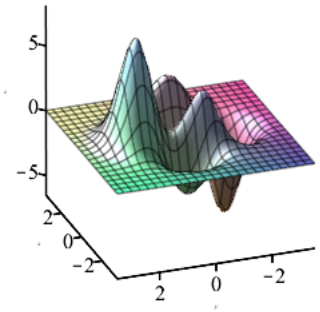
# Surface intéressante

```
> f:=(x,y)->3*(x-1)^2*exp(-x^2-(y+1)^2)-10*(1/5*x-x^3-y^5)*exp(-x^2-
  y^2)-1/3*exp(-(x+1)^2-y^2):
  z=f(x,y);
```

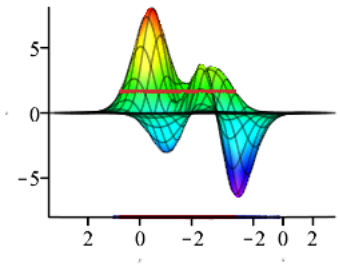
$$z = 3\,(x-1)^2\,e^{-x2-(y+1)^2} - 10\left(\frac{1}{5}\,x - x^3 - y^5\right)e^{-x2-y2} - \frac{e^{-(x+1)2-y2}}{3}$$

```
> Surface:=plot3d([x,y,f(x,y)],x=-3.5..3.5,y=-3.5..3.5,
```

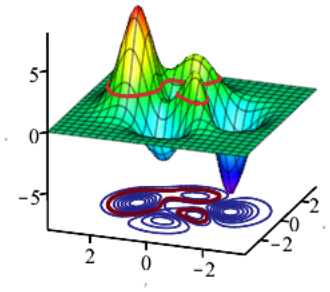
```
                    grid=[80,120],thickness=0):
display(Surface,axes=framed,orientation=[160,65]);
```




```
> Courbes_de_niveau:=contourplot(f(x,y),x=-3.5..3.5,y=-3.5..3.5,
  numpoints=18000,contours=13, color=navy,thickness=0):
> t:=transform((x,y)->[x,y,-8]):
> Courbes_en_évidence:=contourplot(f(x,y),x=-3.5..3.5,y=-3.5..3.5,
  contours=[f(0.75,0.75)],numpoints=9600,thickness=3):
> Niveau_en_évidence:=f(0.75,0.75):
  N:=contourplot3d(f(x,y),x=-3.5..3.5,y=-3.5..3.5,numpoints=9600,
                   contours=[Niveau_en_évidence],
                   color=orange,thickness=3):
> display([Surface,t(Courbes_de_niveau),t(Courbes_en_évidence),N],
  orientation=[-150,90,0],
           lightmodel=light1,shading=ZHUE,axes=framed);
```




```
> display([Surface,t(Courbes_de_niveau),t(Courbes_en_évidence),N],
  orientation=[-160,70,0],
           lightmodel=light1,shading=ZHUE,axes=framed);
```


5
0
-5
2
0
-2
2
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-2