# *Premiers pas en Maple* (Partie 4 de 5)

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Pour voir le contenu des différentes sections, cliquer avec la souris sur le triangle ▶ précédant le titre. La section se déploiera ▼ et son contenu sera affiché.
(un clic gauche ou un clic droit sur le triangle permettra de rétracter la section et son contenu sera alors masqué).

Pour votre confort, vous pouvez ajuster la taille de l'affichage à l'aide de la commande *Facteur de zoom* du menu *Affichage*.

Bonne lecture à tous !

* Ce document Maple est exécutable avec la version 2020.2

## Initialisation

```
> restart;
> with(plots,display,implicitplot3d,spacecurve);
```

$$[display, implicitplot3d, spacecurve] \quad \textbf{(1.1)}$$

## Objectif

Cette quatrième partie a comme objectif d'initier l'élève aux tracés de courbes et de plans dans l'espace. Le tracé de courbes dans l'espace se limitera aux tracés de l'intersections de plans parallèles aux plans de coordonnées et de quadriques.

## Syntaxe paramétrique de tracés de surfaces dans l'espace

Les options permises pour le tracé d'une surface dans l'espace sont trop nombreuses pour être exposées dans ce document. Le lecteur les retrouverons dans l'aide plot3d,options. Tout de même, les nombreux exemples de ce document donneront un bon aperçu de leur utilité. La macro-commande setoptions et setoptions3d de l'extension `plots`, permet de configurer une séquence d'options global pour les tracés 2D et 3D mais deans ce document d'initiation, cette  macro-commande ne sera pas utilisée.

### Surfaces définies explicitement

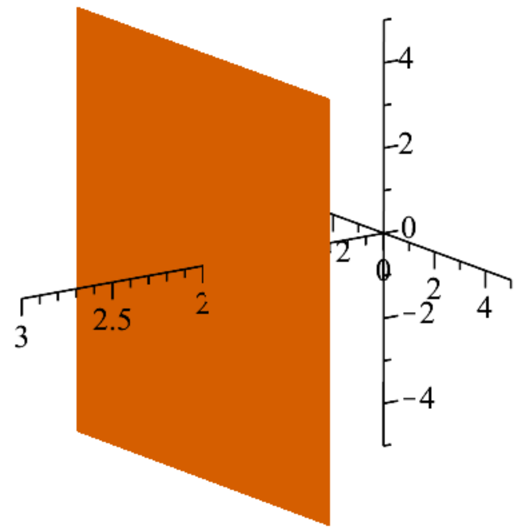
#### *Tracé d'un plan parallèle à deux axes de coordonnées*

Il suffit de préciser une coordonnée constante et de faire varier les deux autres.

Plan d'équation $x = 2$. Fixons l'abscisse et faisons varier l'ordonnée et la cote.

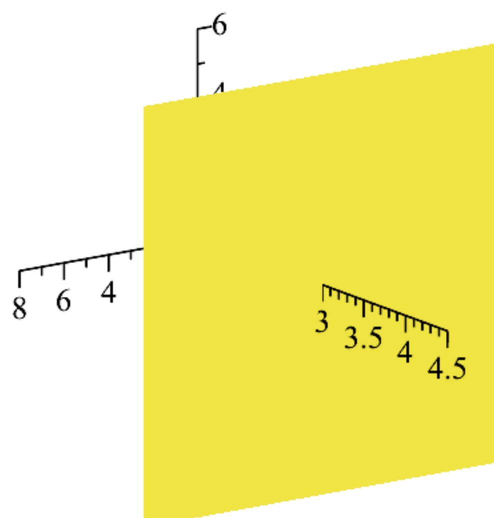
```
> P1:=plot3d([2,y,z],z=-5..5,y=-5..5,
        axes=normal,
        style=patchnogrid,
```

```
        color="Dalton Vermillion",
        lightmodel=none):
P1;
```




Plan d'équation $y = 3$. Fixons l'ordonnée et faisons varier l'abscisse et la cote.

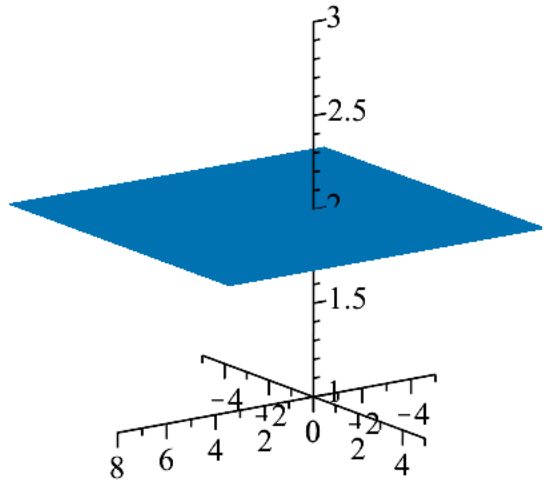
```
> P2:=plot3d([x,3,z],x=-8..8,z=-6..6,
        axes=normal,
        style=patchnogrid,
        color="Dalton Yellow",
        lightmodel=none):
P2;
```




Plan d'équation $z = 2$. Fixons la cote et faisons varier l'ordonnée et la cote.

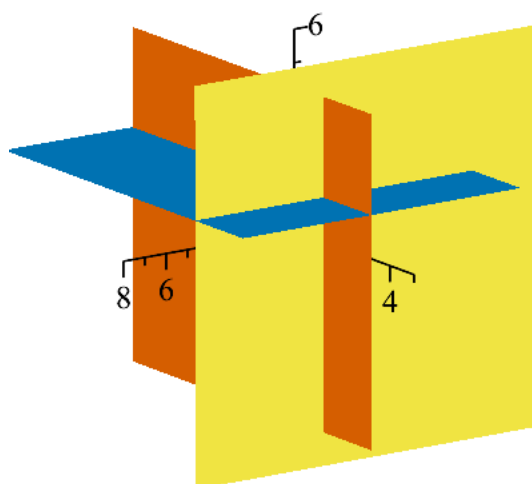
```
> P3:=plot3d([x,y,2],x=-5..8,y=-5..5,
        axes=normal,
        style=patchnogrid,
        color="Dalton Blue",
        lightmodel=none):
```

```
P3;
```




Superposons maintenant ces trois plans dans un même graphique.

```
> display([P1,P2,P3]);
```

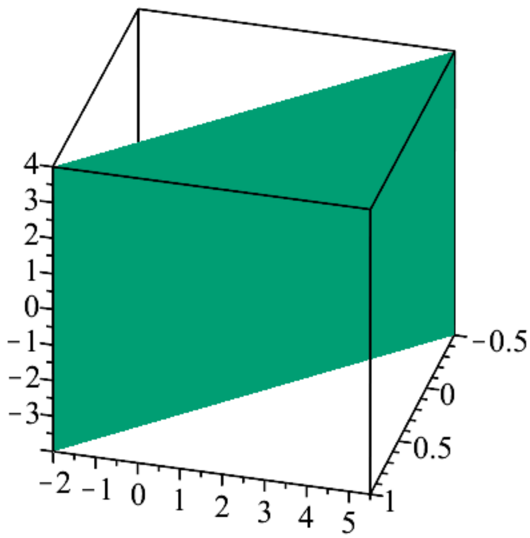



### *Tracé d'un plan parallèle à un axe de coordonnées*

Il suffit de préciser une coordonnée dépendante d'une (seule) autre et faire varier ensuite les deux coordonnées indépendamment.

Plan d'équation $-5x - y + 3 = 0$.

```
> plot3d([x,-5*x+3,z],x=-0.5..1,z=-4..4,
    axes=boxed,lightmodel=none,
    orientation=[15,60],
    style=patchnogrid,
    color="Dalton BluishGreen");
```




### *Tracé d'un plan parallèle à aucun axe de coordonnées*

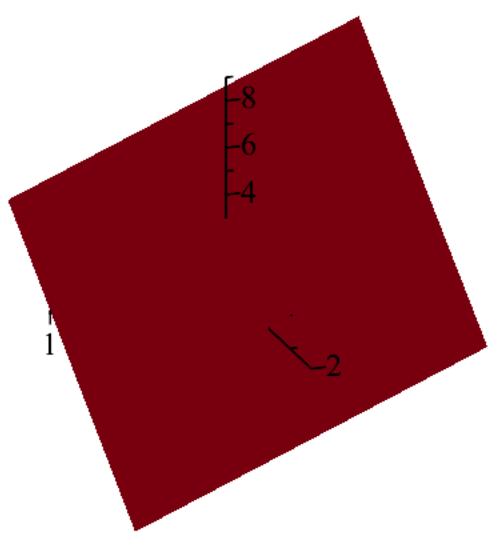
Soit le plan d'équation $x + y + \frac{z}{3} - 1 = 0$.

```
> Éq:=x+y+z/3-1=0;
  isolate(Éq,z);
```

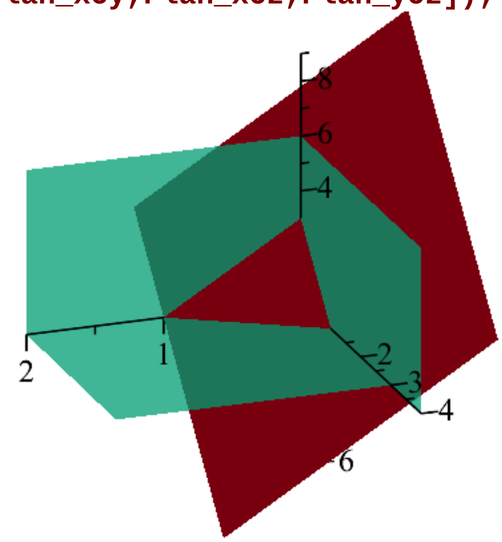
$$Éq := x + y + \frac{z}{3} - 1 = 0$$

$$z = -3x - 3y + 3 \tag{3.1.3.1}$$

```
> Plan:=plot3d([x,y,-3*x-3*y+3],x=-1..1,y=-1 ..2,
          axes=normal,lightmodel=none,
          orientation=[70,70],
          style=surface,
          color="Niagara Burgundy"):
          #transparency=0.05,
          #view=[-1..2,-1..0.75,0..4]
  Plan;
```

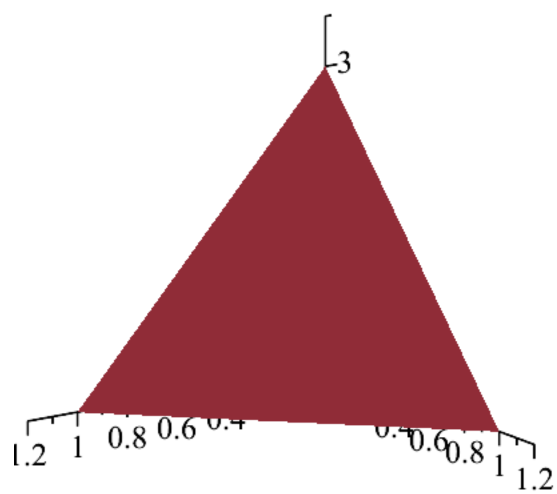


```
> Plan_xOy:=plot3d([x,y,0],x=0..2,y=0 ..3,
        style=patchnogrid,transparency=0.25,
        color="Dalton BluishGreen"):
  Plan_xOz:=plot3d([x,0,z],x=0..2,z=0 ..6,
        style=patchnogrid,transparency=0.25,
        color="Dalton BluishGreen"):
  Plan_yOz:=plot3d([0,y,z],y=0..4,z=0 ..6,
        style=patchnogrid,transparency=0.25,
        color="Dalton BluishGreen"):
  display([Plan,Plan_xOy,Plan_xOz,Plan_yOz]);
```




Visualisons ce plan dans le premier octant.

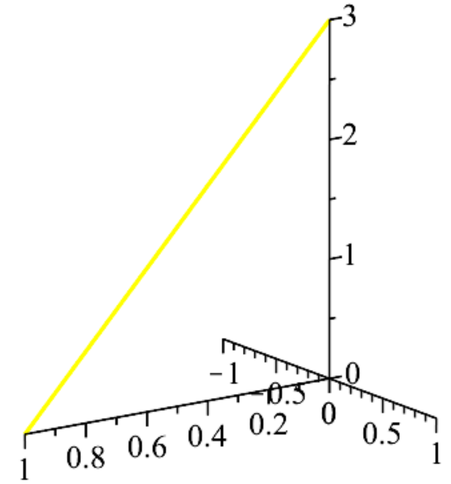
```
> Plan_bis:=plot3d([x,y,-3*x-3*y+3],x=0..1.2,y=0 ..1.2,
        style=patchnogrid,
        color="Niagara Burgundy"):
  display(Plan_bis,view=[0..1.2,0..1.2,0..3.5],axes=normal);
```




Voici comment obtenir la trace de ce plan dans le plan de coordonnés *xOz*.

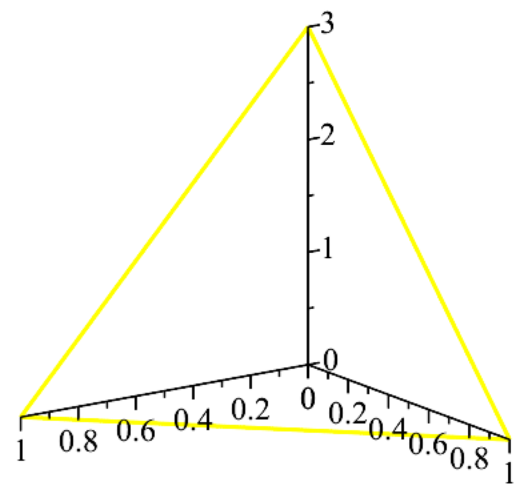
```
> Trace_xOz:=spacecurve([x,0,-3*x+3],x=0..1,thickness=3,color=
```

```
yellow):
display(Trace_xOz,axes=normal);
```

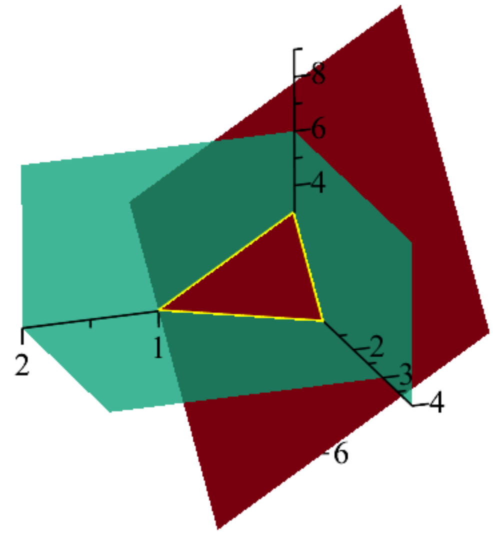



Faisons de même pour les traces dans les deux autres plans de coordonnées et superposons les trois traces avec le plan lui-même.

```
> Trace_xOy:=spacecurve([x,-x+1,0],x=0..1,thickness=3,color=
  yellow):
  Trace_yOz:=spacecurve([0,y,-3*y+3],y=0..1,thickness=3,color=
  yellow):
> display(Trace_xOz,Trace_xOy,Trace_yOz,axes=normal);
```




```
> display([Plan,Plan_xOy,Plan_xOz,Plan_yOz,Trace_xOz,Trace_xOy,
  Trace_yOz],axes=normal);
```



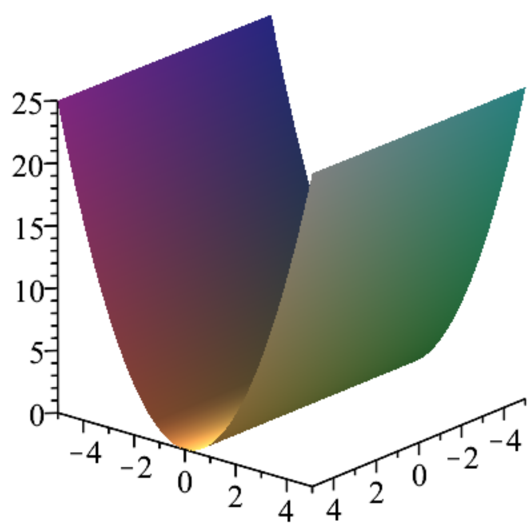

### *Tracé d'une surface cylindrique*

Traçons la surface d'équation $z = y^2$.

```
> f:=(x,y)->y^2;
```

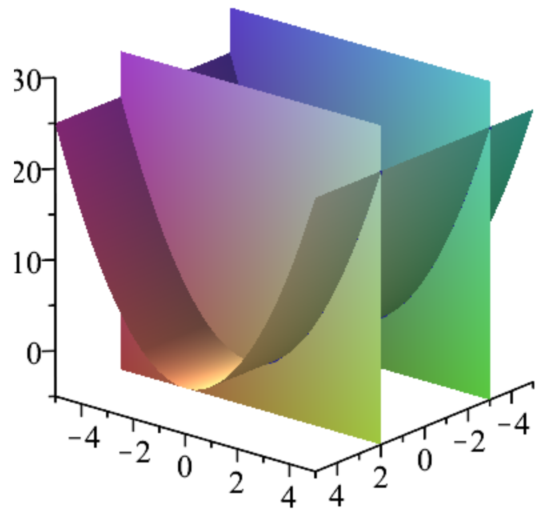
$$f := (x, y) \mapsto y^2$$

```
> Surface:=plot3d([x,y,f(x,y)],x=-5..5,y=-5..5,
            style=patchnogrid,
            axes=framed,
            orientation=[40,70]):
  Surface;
```



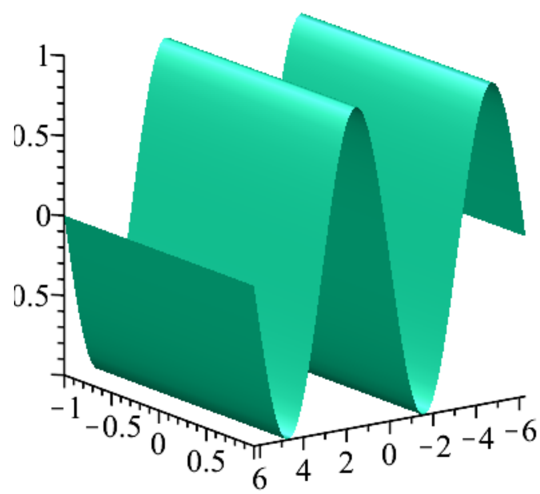

```
> Plan1:=plot3d([2,y,z],y=-5..5,z=-5..30,style=patchnogrid):
  Trace1:=spacecurve([2,y,f(2,y)],y=-5..5,color=navy,thickness=2):
  Plan2:=plot3d([-3,y,z],y=-5..5,z=-5..30,style=patchnogrid):
  Trace2:=spacecurve([-3,y,f(-3,y)],y=-5..5,color=navy,thickness=
  2):
> Surface:=plot3d([x,y,f(x,y)],x=-5..5,y=-5..5,style=patchnogrid):
  display(Plan1,Plan2,Trace1,Trace2,Surface,axes=framed,
```

```
orientation=[40,70]);
```

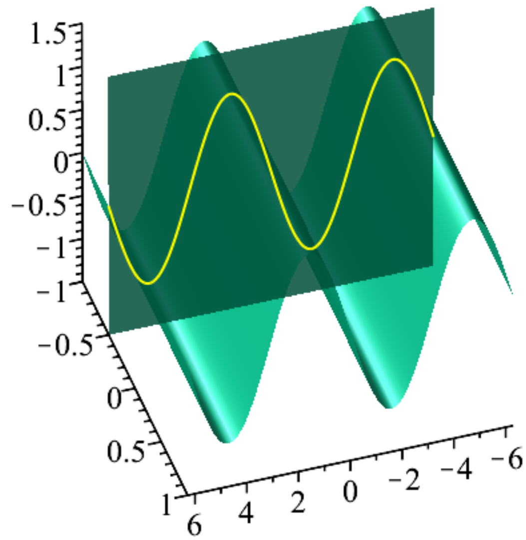



```
> f:=(x,y)->sin(x);
  Surface:=plot3d([x,y,f(x,y)],x=-2*Pi..2*Pi,y=-1..1,style=
  surface,grid=[80,60]):
  display(Surface,axes=framed,lightmodel=light3,color="Dalton
  BluishGreen");
```

$$f := (x, y) \mapsto \sin(x)$$




```
> Surface:=plot3d([x,y,f(x,y)],x=-2*Pi..2*Pi,y=-1..1,style=
  surface,grid=[80,60],transparency=0.05):
  Plan1:=plot3d([x,-0.5,z],x=-2*Pi..2*Pi,z=-1.5..1.5,style=
  surface,lightmodel=light3,transparency=0.15):
  Trace1:=spacecurve([x,-0.5,f(x,y)],x=-2*Pi..2*Pi,thickness=2,
  color=yellow):
  display([Surface,Plan1,Trace1],axes=framed,orientation=[72,49],
  color="Dalton BluishGreen");
```




### *Tracé d'un paraboloïde elliptique*

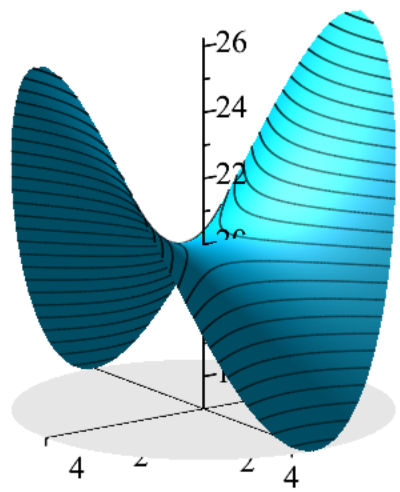
Traçons le paraboloïde d'équation $z = 20 + \frac{x^2}{4} - \frac{y^2}{5}$.

```
> f:=(x,y)->20+x^2/4-y^2/5;
```

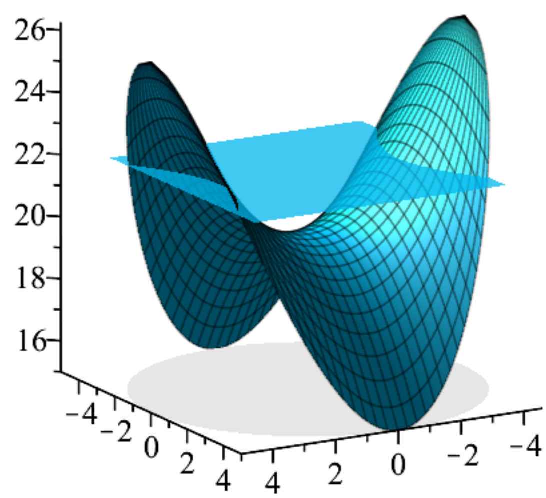
$$f := (x, y) \mapsto 20 + \frac{x^2}{4} - \frac{y^2}{5}$$

```
> Lieu:=plot3d([x,y,f(x,y)],x=-5..5,y=-sqrt(25-x^2)..sqrt(25-x^2),
             color="Bright Cyan",
             grid=[160,120]):
  Domaine:=plot3d([x,y,f(0,5)],x=-5..5,y=-sqrt(25-x^2)..sqrt(25-
  x^2),
           grid=[80,20],
           style=patchnogrid,
           color=grey):
```

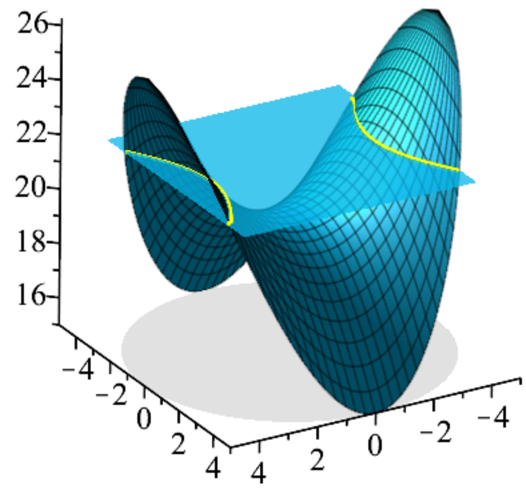
```
> display(Lieu,Domaine,
        axes=normal,
        style=surfacecontour,contours = 25,
        orientation=[60,50,0]);
```



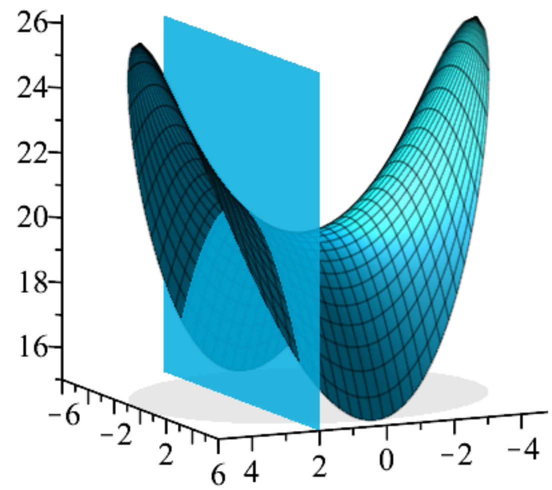

```
> Plan1:=plot3d([x,y,22],x=-4..4,y=-4..4,style=surface,color=
  "Bright Cyan",transparency=0.25):
  display([Lieu,Domaine,Plan1],
          axes=framed,
          orientation=[60,75,0]);
```



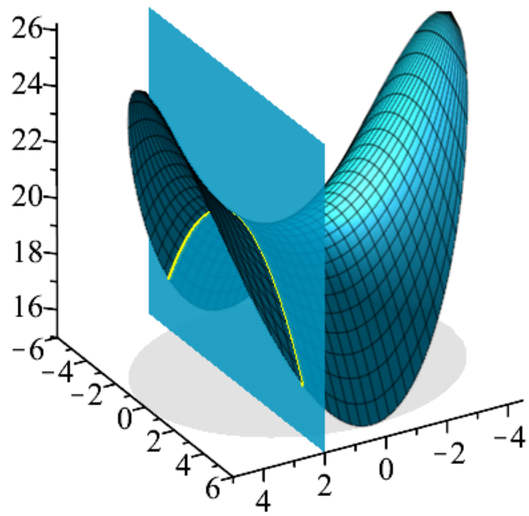

```
> Trace1:=spacecurve([x,1/2*sqrt(-40+5*x^2),22,x=sqrt(8)+10E-6..
  sqrt(8)+1.1],thickness= 4,color=yellow):
  Trace2:=spacecurve([x,-1/2*sqrt(-40+5*x^2),22,x=sqrt(8)+10E-6..
  sqrt(8)+1.1],thickness=4,color=yellow):
  Trace3:=spacecurve([x,1/2*sqrt(-40+5*x^2),22,x=-sqrt(8)-10E-6..-
  sqrt(8)-1.1],thickness= 4,color=yellow):
  Trace4:=spacecurve([x,-1/2*sqrt(-40+5*x^2),22,x=-sqrt(8)-10E-6..
  -sqrt(8)-1.1],thickness=4,color=yellow):
  display([Domaine,Lieu,Plan1,Trace||(1..4)],
          axes=framed,
          orientation=[60,65,0]);
```



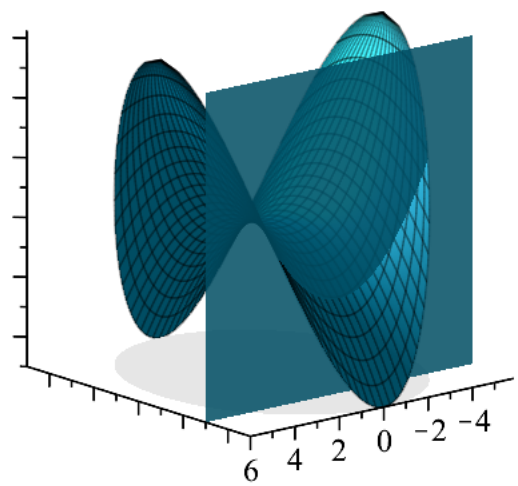

```
> Plan2:=plot3d([2,y,z],y=-6..6,z=15..26,style=surface,color=
  "Bright Cyan",transparency=0.15):
  display([Lieu,Domaine,Plan2],
           axes=framed,
           orientation=[65,80,0]);
```



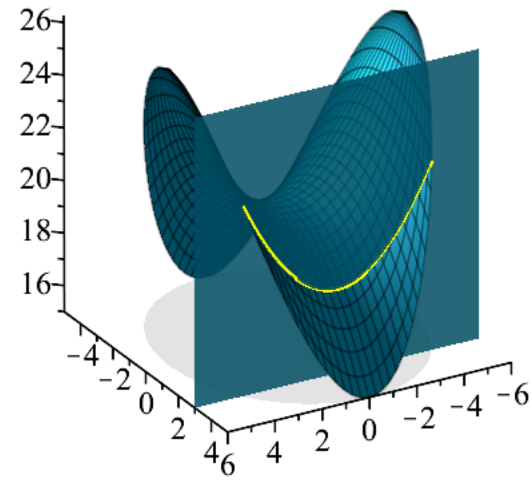

```
> Trace_yOz:=spacecurve([2,y,21-1/5*y^2,y=-sqrt(25-2^2)..sqrt(25
  -2^2)],
             thickness=4,color=yellow):
  display([Lieu,Domaine,Plan2,Trace_yOz],
           axes=framed,
           orientation=[60,63]);
```




```
> Plan3:=plot3d([x,3,z],x=-6..6,z=15..26,style=surface,color=
  "Bright Cyan",transparency=0.15):
  display([Lieu,Domaine,Plan3],
            axes=framed,
            orientation=[50,75]);
```




```
> Trace_xOz:=spacecurve([x,3,91/5+1/4*x^2,x=-4..4],thickness=4,
  color=yellow):
  display([Lieu,Domaine,Plan3,Trace_xOz],
            axes=framed,
            orientation=[60,65,0]);
```

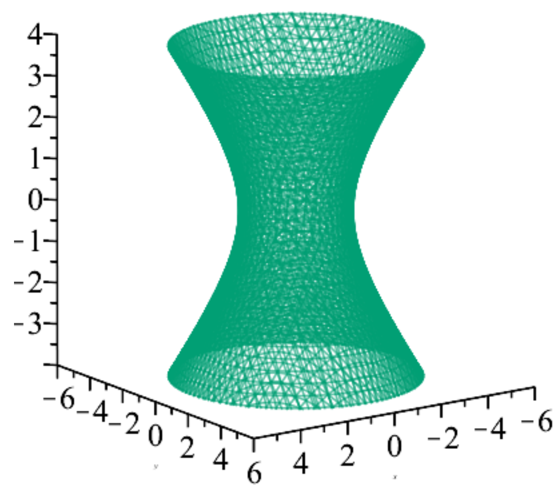



## Surfaces définies implicitement

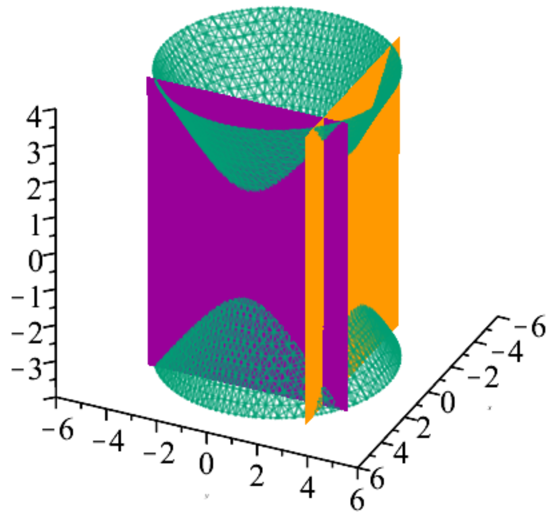
### *Tracé d'un hyperboloïde à une nappe*

Traçons la surface hyperbolique à deux nappes d'équation $x^2 + y^2 - z^2 = 4$.

```
> Lieu:=implicitplot3d(x^2+y^2-z^2=4,x=-5..5,y=-5..5,z=-5..5,grid=
[40,40,40],
        view=[-6..6,-6..6,-4..4],style=wireframe,axes=framed,
color="Dalton BluishGreen"):
Lieu;
```




```
> Plan1:=plot3d([2.5,y,z],y=-4..4,z=-4..4,style=patchnogrid,color=
"Bright Purple"):
Plan2:=plot3d([x,3,z],x=-4..4,z=-4..4,style=patchnogrid,color=
"Bright Orange"):
display([Lieu,Plan1,Plan2],orientation=[25,60,0],lightmodel=
none);
```

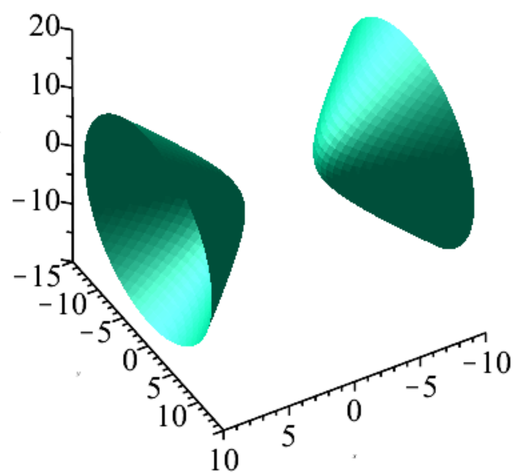



*Tracé d'un hyperboloïde à deux nappes*

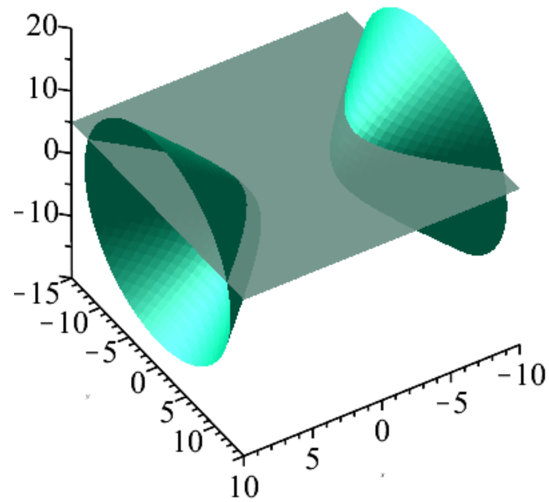
Traçons la surface hyperbolique à deux nappes d'équation $\frac{x^2}{9} - \frac{y^2}{16} - \frac{z^2}{25} = 1$.

```
> Éq_Lieu:=x^2/9-y^2/16-z^2/25=1;
  Lieu:=implicitplot3d(Éq_Lieu,x=-10..10,y=-15..15,z=-20..20,
        grid=[40,40,40],
        style=patchnogrid,
        axes=framed):
  display(Lieu,orientation=[60,50],color="Dalton BluishGreen");
```

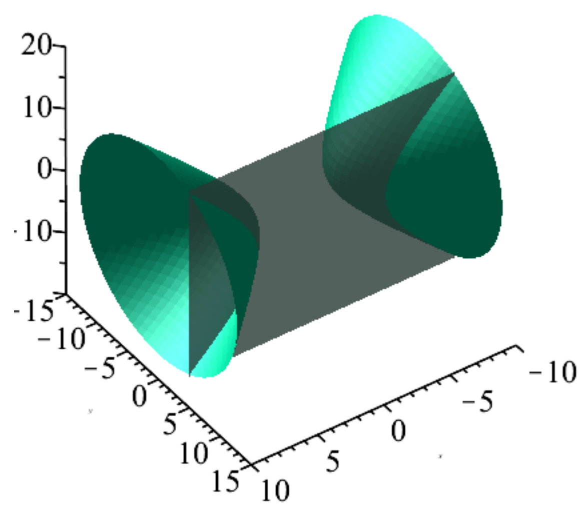
$$\mathit{Éq\_Lieu} := \frac{x^2}{9} - \frac{y^2}{16} - \frac{z^2}{25} = 1$$



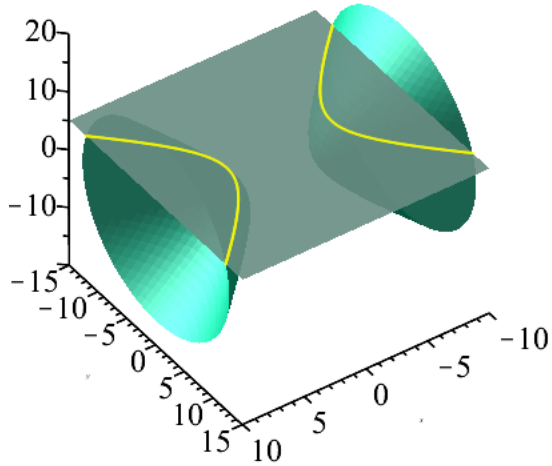

```
> Plan1:=plot3d([x,y,5],x=-10..10,y=-15..15,style=patchnogrid,
  color="Nautical PaleGreen",transparency=0.10):
  display([Lieu,Plan1],orientation=[58,50],color="Dalton
  BluishGreen");
```



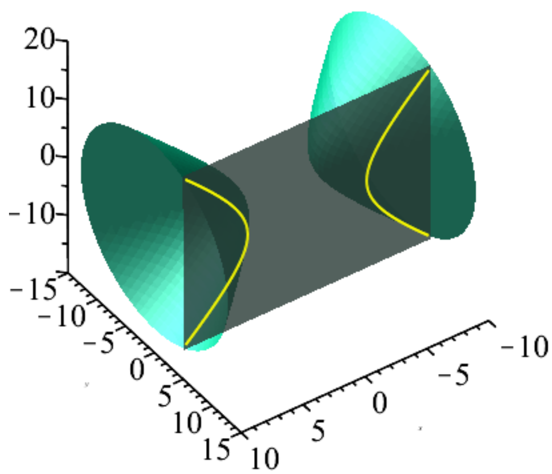

```
> Plan2:=plot3d([x,5,z],x=-10..10,z=-15..15,style=patchnogrid,
  color="Nautical PaleGreen",transparency=0.20):
  display([Lieu,Plan2],orientation=[55,50,0],color="Dalton
  BluishGreen");
```




```
> Trace1:=spacecurve([t,4/3*sqrt(t^2-18),5],t=sqrt(18)+10E-6..sqrt
  (18)+5.8,color=yellow,thickness=2):
  Trace2:=spacecurve([t,-4/3*sqrt(t^2-18),5],t=sqrt(18)+10E-6..
  sqrt(18)+5.8,color=yellow,thickness=2):
  Trace3:=spacecurve([t,4/3*sqrt(t^2-18),5],t=-sqrt(18)-5.8..-sqrt
  (18)-10E-6,color=yellow,thickness=2):
  Trace4:=spacecurve([t,-4/3*sqrt(t^2-18),5],t=-sqrt(18)-5.8..-
  sqrt(18)-10E-6,color=yellow,thickness=2):
> display([Lieu,Plan1,Trace||(1..4)],orientation=[55,50],color=
  "Dalton BluishGreen",transparency=0.10);
```




```
> Trace1:=spacecurve([t,5,5/12*sqrt(16*t^2-369)],t=sqrt(369/16)
  +10E-6..sqrt(369/16)+5,color=yellow,thickness=2):
  Trace2:=spacecurve([t,5,-5/12*sqrt(16*t^2-369)],t=sqrt(369/16)
  +10E-6..sqrt(369/16)+5,color=yellow,thickness=2):
  Trace3:=spacecurve([t,5,5/12*sqrt(16*t^2-369)],t=-sqrt(369/16)
  -5..-sqrt(369/16)-10E-6,color=yellow,thickness=2):
  Trace4:=spacecurve([t,5,-5/12*sqrt(16*t^2-369)],t=-sqrt(369/16)
  -5..-sqrt(369/16)-10E-6,color=yellow,thickness=2):
> display([Lieu,Plan2,Trace||(1..4)],orientation=[55,50,0],color=
  "Dalton BluishGreen",transparency=0.10);
```

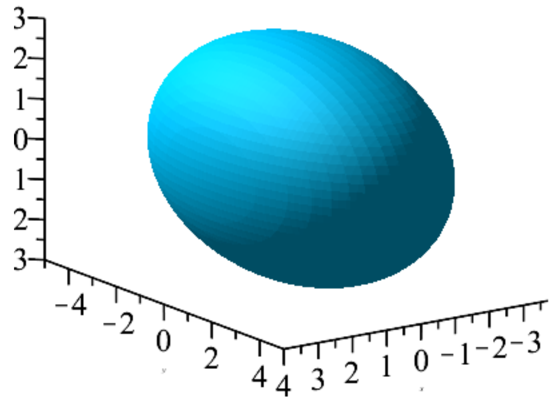



### *Tracé d'un ellipsoïde*

Soit l'ellipsoïde d'équation $\frac{x^2}{8}+\frac{y^2}{25}+\frac{z^2}{9}-1=0$.

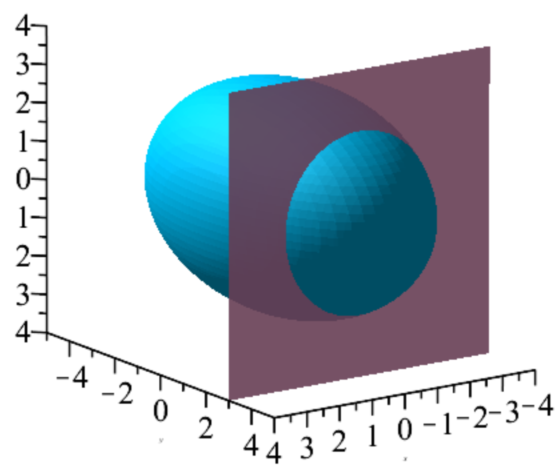
```
> Ellipsoïde:=implicitplot3d(x^2/8+1/25*y^2+1/9*z^2-1 = 0,x=-4..4,
  y=-5..5,z=-3..3,
        style=patchnogrid,grid=[30,60,30]):
> display(Ellipsoïde,axes=framed,color="Bright Cyan",glossiness=
```

```
0.3,scaling=constrained,orientation=[55,75,0]);
```

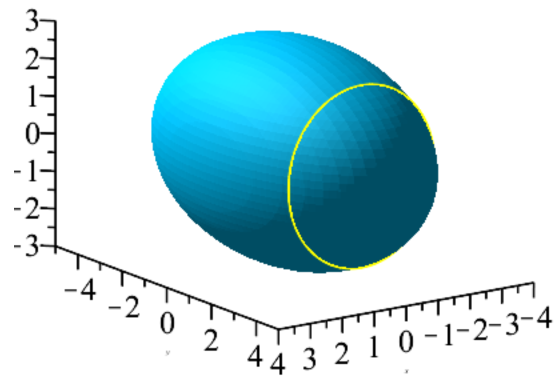



Obtenons maintenant les traces de trois plans parallèles aux plans de coordonnées sur cet ellipsoïde.

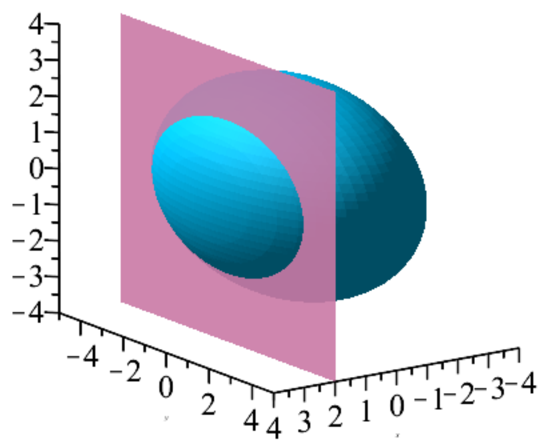
```
> Plan_xOz:=plot3d([x,3,z],x=-4..4,z=-4..4,style=patchnogrid,
  color="Dalton ReddishPurple",transparency=0.10):
  display(Ellipsoïde,Plan_xOz,axes=framed,color="Bright Cyan",
  glossiness=0.3,scaling=constrained);
```



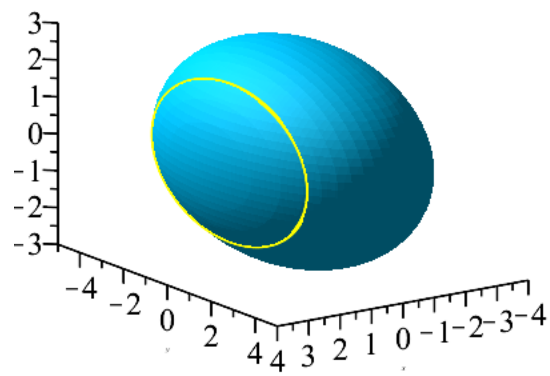

```
> Trace1:=spacecurve([sqrt(8*16/25)*cos(t),3,sqrt(9*16/25)*sin(t)
  ],t=0..2*Pi,color=yellow,thickness=3):
  display(Ellipsoïde,Trace1,axes=framed,color="Bright Cyan",
  glossiness=0.3,scaling=constrained);
```



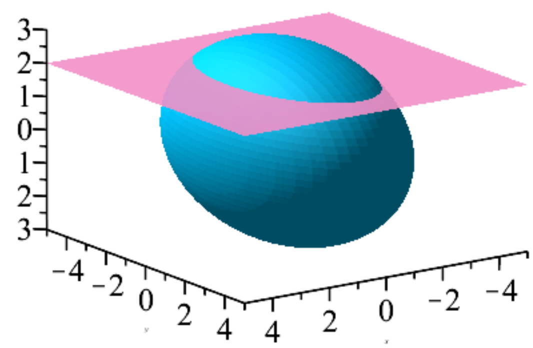

```
> Plan_yOz:=plot3d([2,y,z],y=-5..5,z=-4..4,style=patchnogrid,
  color="Dalton ReddishPurple",transparency=0.10):
  display(Ellipsoïde,Plan_yOz,axes=framed,color="Bright Cyan",
  glossiness=0.3,scaling=constrained);
```



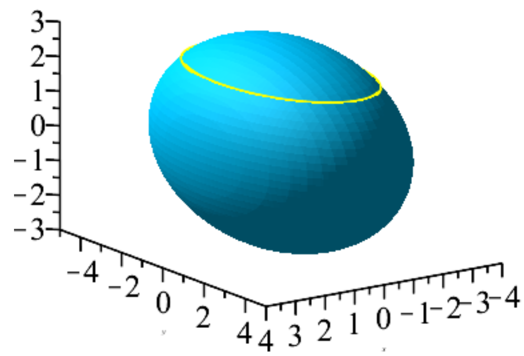

```
> Trace2:=spacecurve([2,sqrt(25/2)*cos(t),sqrt(9/2)*sin(t)],t=0.
  .2*Pi,color=yellow,thickness=3):
  display(Ellipsoïde,Trace2,axes=framed,color="Bright Cyan",
  glossiness=0.3,scaling=constrained);
```




```
> Plan_xOy:=plot3d([x,y,2],x=-5..5,y=-5..5,style=patchnogrid,
  color="Dalton ReddishPurple",transparency=0.10):
  display(Ellipsoïde,Plan_xOy,axes=framed,color="Bright Cyan",
  glossiness=0.3,scaling=constrained);
```

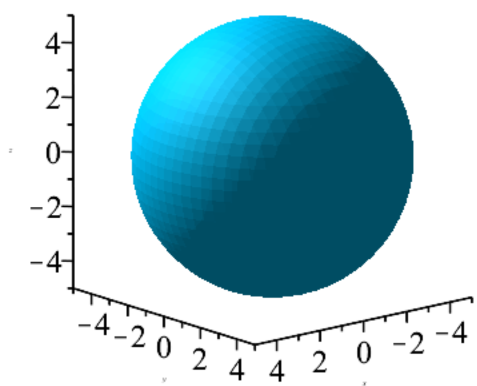



```
> Trace3:=spacecurve([sqrt(8*5/9)*cos(t),sqrt(25*5/9)*sin(t),2],t=
  0..2*Pi,color=yellow,thickness=3):
  display(Ellipsoïde,Trace3,axes=framed,color="Bright Cyan",
  glossiness=0.3,scaling=constrained);
```



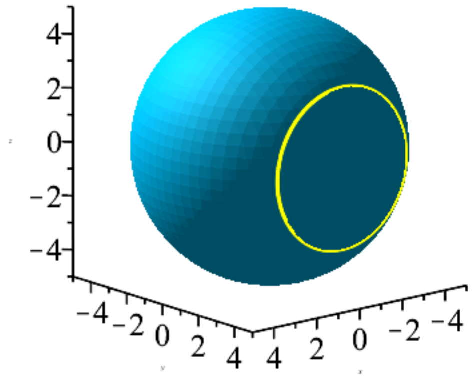
### *Tracé d'une sphère*

```
> Sphère:=implicitplot3d(x^2+y^2+z^2 = 25,x=-5..5,y=-5..5,z=-5..5,
  style=patchnogrid,grid=[25,25,25],
  axes=normal,scaling=constrained,axes=framed,color="Bright Cyan",
  glossiness=0.3,orientation=[50,75,0]);
```



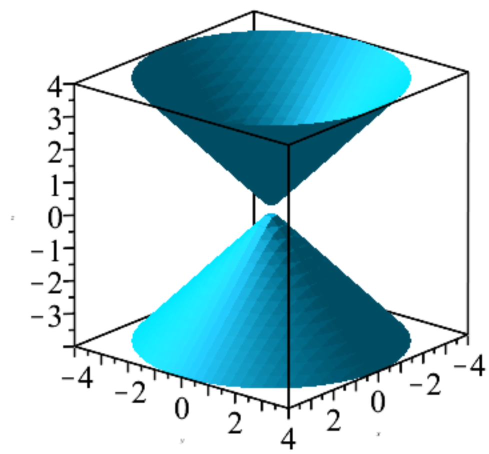
Illustrons la trace sur la sphère du plan $y = 4$.

```
> Trace1:=spacecurve([3*cos(t),4,3*sin(t)],t=0..2*Pi,color=yellow,
  thickness=3):
  display(Sphère, Trace1);
```

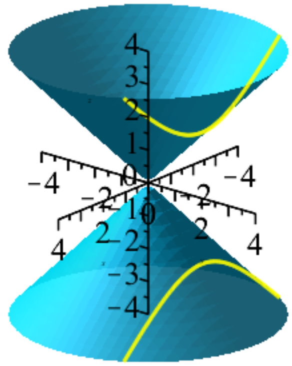



*Tracé d'un cône elliptique*

```
> Cône:=implicitplot3d(x^2+y^2-z^2 = 0,x=-4..4,y=-4..4,z=-4..4,
  style=patchnogrid,grid=[30,30,30],
  orientation=[40,70],color="Bright Cyan",glossiness=0.3);
```




```
> Trace1:=spacecurve([2*tan(t),2,-2/cos(t)],t=-Pi/3..Pi/3,color=
  yellow,thickness=3):
  Trace2:=spacecurve([2*tan(t),2,2/cos(t)],t=-Pi/3..Pi/3,color=
  yellow,thickness=3):
  display(Cône, Trace1,Trace2,transparency=0.1,axes=normal,
  scaling=constrained);
```


4
3
2
1
0
−1
−2
−3
−4
−4
−2
2
4
−4
−2
2
4