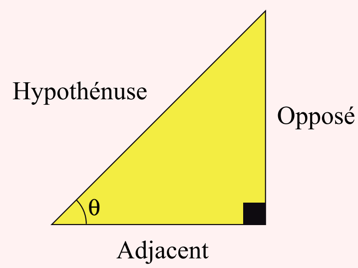
Trigonométrie rectiligne

$$\sin(\theta) = \frac{\text{Opp}}{\text{Hyp}} \qquad \csc(\theta) = \frac{\text{Hyp}}{\text{Opp}}$$

$$\cos(\theta) = \frac{\text{Adj}}{\text{Hyp}} \qquad \sec(\theta) = \frac{\text{Hyp}}{\text{Adj}}$$

$$\tan(\theta) = \frac{\text{Opp}}{\text{Adj}} \qquad \cot(\theta) = \frac{\text{Adj}}{\text{Opp}}$$



Relations inverses

$$\csc(x) \equiv \frac{1}{\sin(x)} \qquad \sec(x) \equiv \frac{1}{\cos(x)} \qquad \cot(x) \equiv \frac{1}{\tan(x)}$$

Relations quotients

$$\tan(x) \equiv \frac{\sin(x)}{\cos(x)} \qquad \cot(x) \equiv \frac{\cos(x)}{\sin(x)}$$

Relations cofonctions

$$\sin\left(\frac{\pi}{2} - t\right) \equiv \cos(t) \qquad \cos\left(\frac{\pi}{2} - t\right) \equiv \sin(t)$$

$$\tan\left(\frac{\pi}{2} - t\right) \equiv \cot(t) \qquad \cot\left(\frac{\pi}{2} - t\right) \equiv \tan(t)$$

$$\sec\left(\frac{\pi}{2} - t\right) \equiv \csc(t) \qquad \csc\left(\frac{\pi}{2} - t\right) \equiv \sec(t)$$

Relations pythagoriciennes

$$\sin^2(t) + \cos^2(t) \equiv 1$$

$$1 + \tan^2(t) \equiv \sec^2(t)$$

$$1 + \cot^2(t) \equiv \csc^2(t)$$

Triangle obliquangle (triangle acutangle ou triangle obtusangle)

Loi des sinus

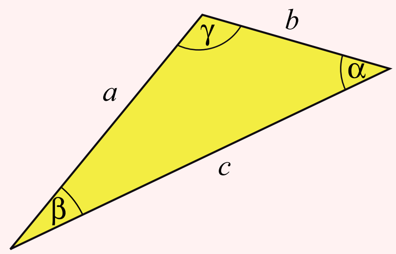
$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{c}$$

Loi des cosinus

$$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$$

$$b^2 = a^2 + c^2 - 2ac\cos(\beta)$$

$$c^2 = a^2 + b^2 - 2ab\cos(\gamma)$$



Diverses mesures d'angles

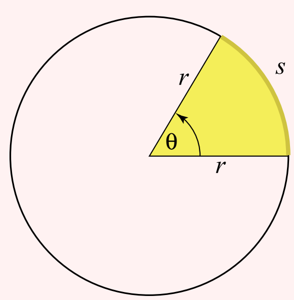
π radians $= 180°$

$s = r\theta$

$A = \frac{1}{2}r^2\theta \quad (\theta \text{ en radians})$

$$\theta \times \frac{\pi}{180} = \alpha \text{ radians}$$

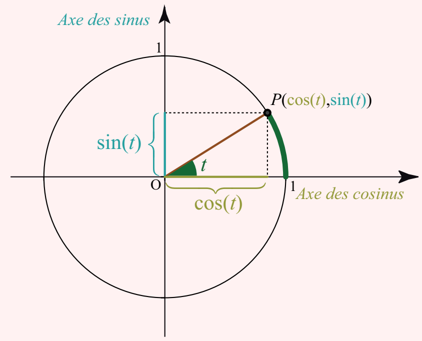
$$\alpha \text{ radians} \times \frac{180}{\pi} = \theta$$



Trigonométrie circulaire

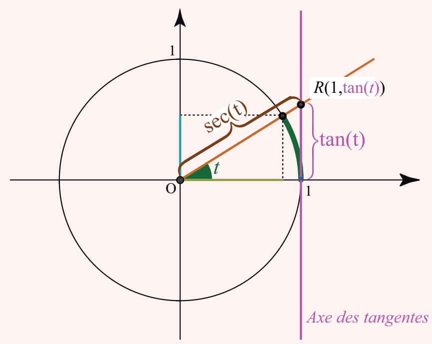
$\cos(t)$ correspond à l'abscisse du point $P(t)$

$\sin(t)$ correspond à l'ordonnée du point $P(t)$



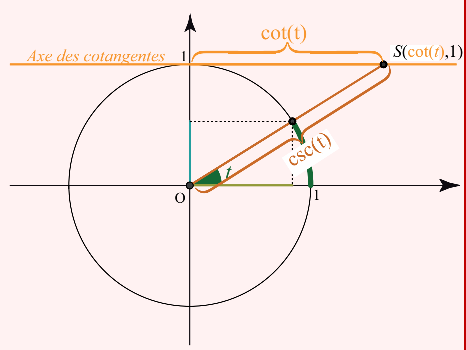
$\tan(t)$ correspond à l'ordonnée du point $R(t)$

$\sec(t)$ correspond à la mesure algébrique du segment $OR(t)$



$\cot(t)$ correspond à l'abscisse du point $S(t)$

$\csc(t)$ correspond à la mesure algébrique du segment $OS(t)$



Remarque: La $\sec(t)$ définie par la mesure algébrique du segment $OR(t)$ correspond à la longueur du segment $OR(t)$ affectée d'un signe positif si le prolongement du segment $OP(t)$ jusqu'à l'axe des tangentes pour obtenir le point d'intersection $R(t)$ s'effectue sans changement de quadrant. Sinon, cette longueur sera affectée d'un signe négatif.

Il en est de même pour la $\csc(t)$.

Identités trigonométriques remarquables

$$\text{Identités d'angles composés}\begin{cases}\sin(u \pm v) \equiv \sin(u)\cos(v) \pm \sin(v)\cos(u)\\ \cos(u \pm v) \equiv \cos(u)\cos(v) \mp \sin(u)\sin(v)\\ \tan(u \pm v) \equiv \dfrac{\tan(u) \pm \tan(v)}{1 \mp \tan(u)\tan(v)}\end{cases}$$

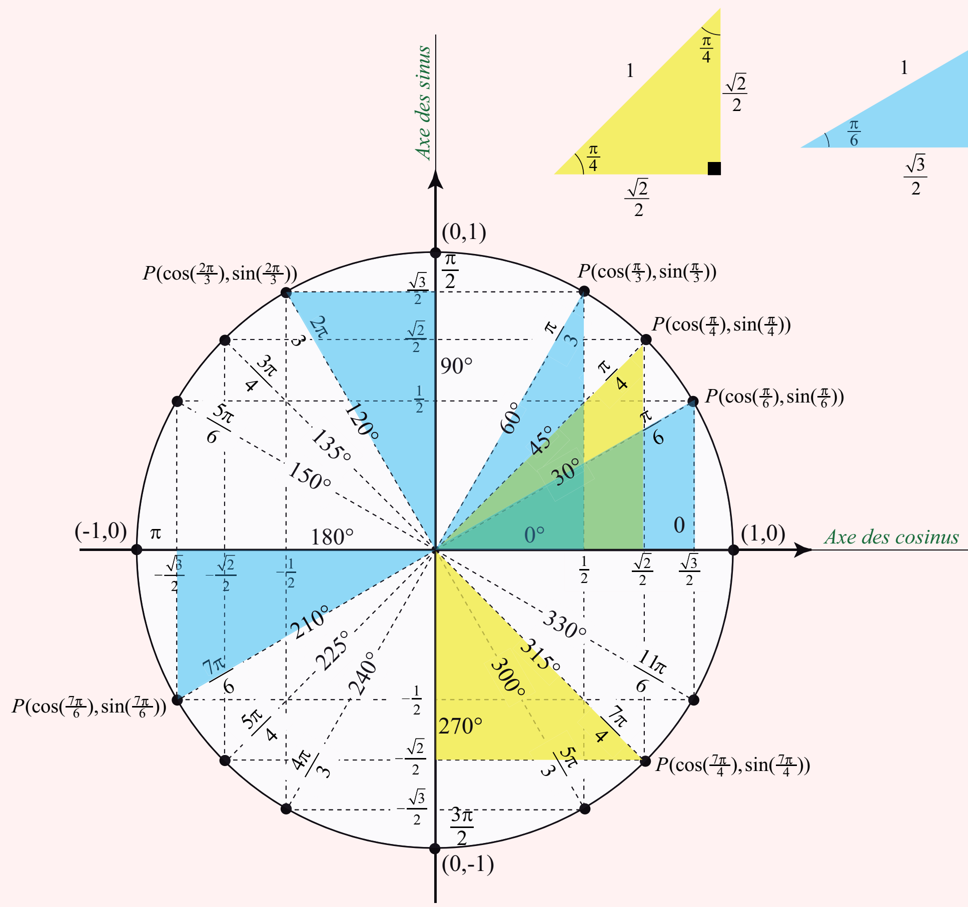
Transformation des sommes en produits

$$\begin{cases}\sin(u+v) + \sin(u-v) \equiv 2\sin(u)\cos(v)\\ \sin(u+v) - \sin(u-v) \equiv 2\cos(u)\sin(v)\\ \cos(u+v) + \cos(u-v) \equiv 2\cos(u)\cos(v)\\ \cos(u+v) - \cos(u-v) \equiv -2\sin(u)\sin(v)\end{cases}$$

Identités d'angles multiples

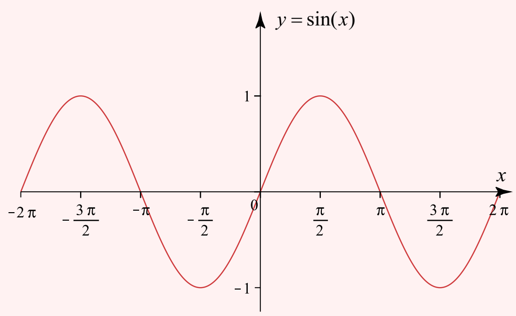
$$\begin{cases}\sin(2t) \equiv 2\sin(t)\cos(t)\\ \cos(2t) \equiv \cos^2(t) - \sin^2(t)\\ \tan(2t) \equiv \dfrac{2\tan(t)}{1-\tan^2(t)}\\ \sin^2(t) \equiv \dfrac{1-\cos(2t)}{2}\\ \cos^2(t) \equiv \dfrac{1+\cos(2t)}{2}\\ \sin\left(\dfrac{t}{2}\right) \equiv \pm\sqrt{\dfrac{1-\cos(t)}{2}}\\ \cos\left(\dfrac{t}{2}\right) \equiv \pm\sqrt{\dfrac{1+\cos(t)}{2}}\\ \tan\left(\dfrac{t}{2}\right) \equiv \dfrac{\sin(t)}{1+\cos(t)}\\ \tan\left(\dfrac{t}{2}\right) \equiv \dfrac{1-\cos(t)}{\sin(t)}\end{cases}$$

Trigonométrie circulaire et triangles remarquables

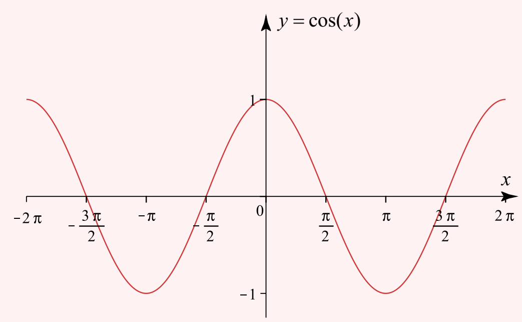


	α	0° / 0	30° / $\frac{\pi}{6}$	45° / $\frac{\pi}{4}$	60° / $\frac{\pi}{3}$	90° / $\frac{\pi}{2}$	180° / π	270° / $\frac{3\pi}{2}$
	sin(α)	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	−1
	cos(α)	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	1	0
$\frac{\sin(\alpha)}{\cos(\alpha)} =$	tan(α)	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$		0	
$\frac{\cos(\alpha)}{\sin(\alpha)} = \frac{1}{\tan(\alpha)} =$	cot(α)		$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0		0
$\frac{1}{\cos(\alpha)} =$	sec(α)	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2		1	
$\frac{1}{\sin(\alpha)} =$	csc(α)		2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$			−1

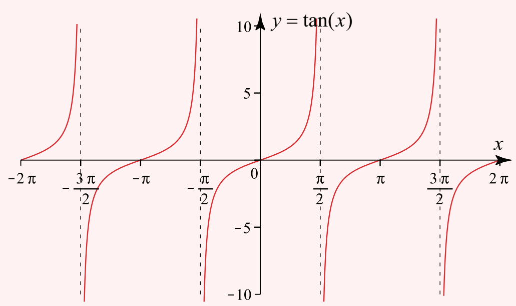
Graphiques des fonctions trigonométriques



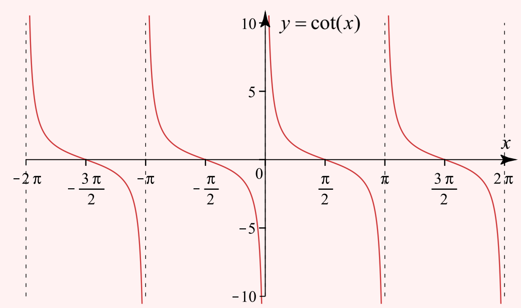
$$\begin{aligned}\text{Dom sin} &= \mathbb{R}\\ \text{Ima sin} &= [-1, 1]\end{aligned}$$



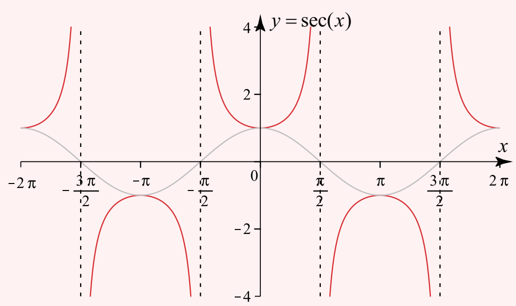
$$\begin{aligned}\text{Dom cos} &= \mathbb{R}\\ \text{Ima cos} &= [-1, 1]\end{aligned}$$



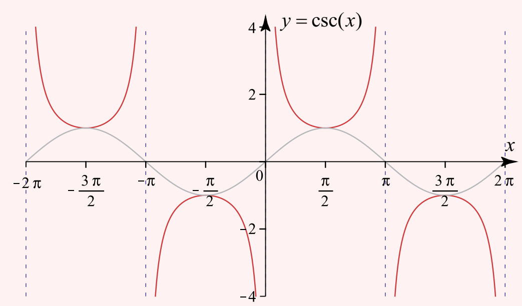
$$\begin{aligned}\text{Dom tan} &= \mathbb{R}\backslash\{\tfrac{\pi}{2} + k\pi, k \in \mathbb{Z}\}\\ \text{Ima tan} &= \mathbb{R}\end{aligned}$$



$$\begin{aligned}\text{Dom cot} &= \mathbb{R}\backslash\{k\pi, k \in \mathbb{Z}\}\\ \text{Ima cot} &=]-\infty, -1] \cup [1, \infty[\end{aligned}$$

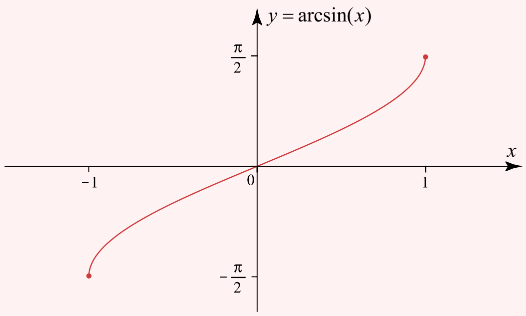


$$\begin{aligned}\text{Dom sec} &= \mathbb{R}\backslash\{\tfrac{\pi}{2} + k\pi, k \in \mathbb{Z}\}\\ \text{Ima sec} &=]-\infty, -1] \cup [1, \infty[\end{aligned}$$

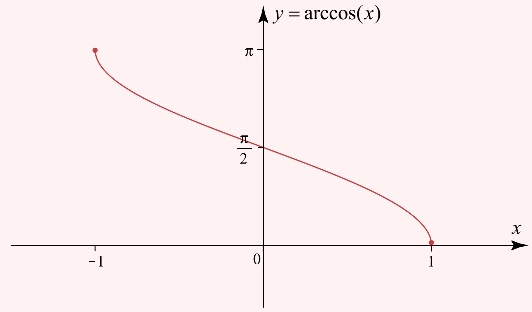


$$\begin{aligned}\text{Dom csc} &= \mathbb{R}\backslash\{k\pi, k \in \mathbb{Z}\}\\ \text{Ima csc} &=]-\infty, -1] \cup [1, \infty[\end{aligned}$$

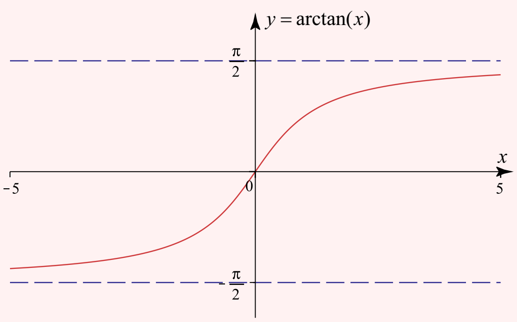
Graphiques des fonctions trigonométriques réciproques



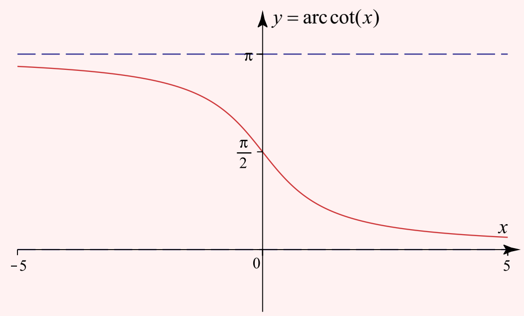
$$\begin{aligned}\text{Dom arcsin} &= [-1, 1]\\ \text{Ima arcsin} &= \left[-\tfrac{\pi}{2}, \tfrac{\pi}{2}\right]\end{aligned}$$



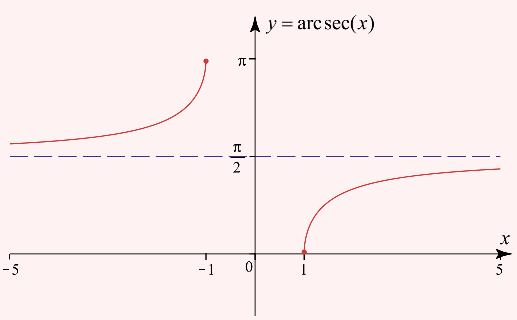
$$\begin{aligned}\text{Dom arccos} &= [-1, 1]\\ \text{Ima arccos} &= [0, \pi]\end{aligned}$$



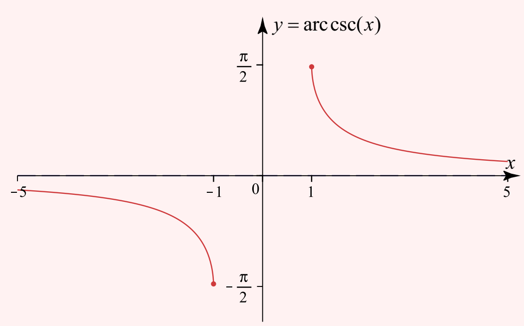
$$\begin{aligned}\text{Dom arctan} &= \mathbb{R}\\ \text{Ima arctan} &= \left]-\tfrac{\pi}{2}, \tfrac{\pi}{2}\right[\end{aligned}$$



$$\begin{aligned}\text{Dom arccot} &= \mathbb{R}\\ \text{Ima arccot} &=]0, \pi[\end{aligned}$$

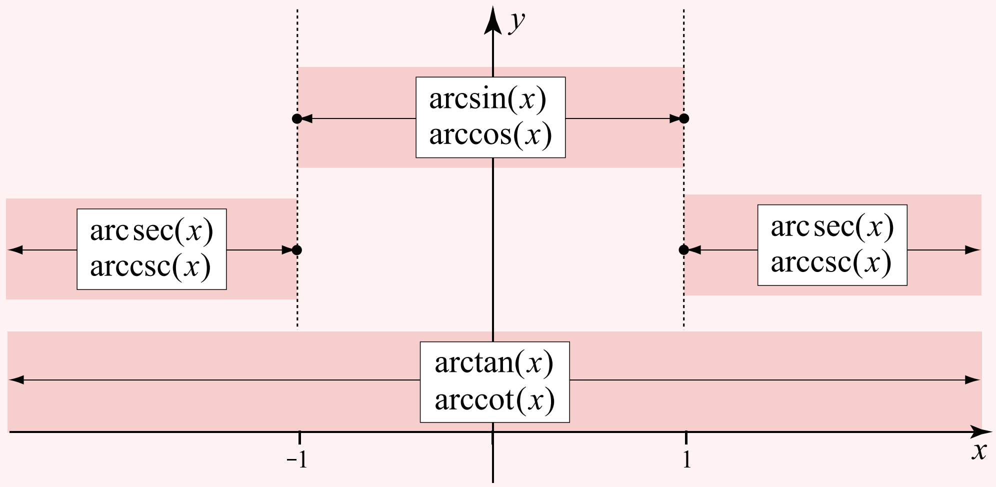


$$\begin{aligned}\text{Dom arcsec} &=]-\infty, -1] \cup [1, \infty[\\ \text{Ima arcsec} &= [0, \pi] \backslash \left\{\tfrac{\pi}{2}\right\}\end{aligned}$$

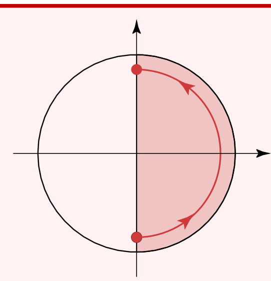
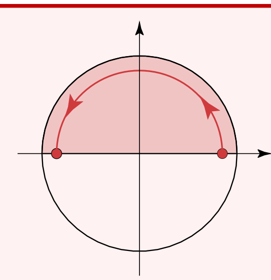
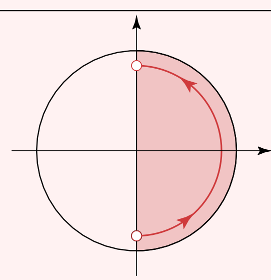
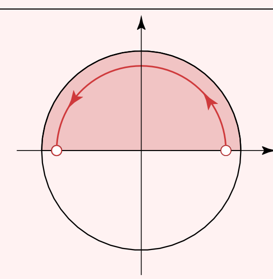
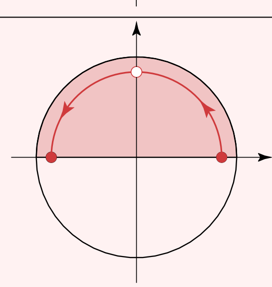
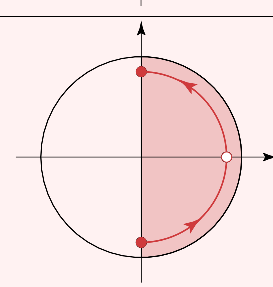


$$\begin{aligned}\text{Dom arccsc} &=]-\infty, -1] \cup [1, \infty[\\ \text{Ima arccsc} &= \left[-\tfrac{\pi}{2}, \tfrac{\pi}{2}\right] \backslash \{0\}\end{aligned}$$

Domaines des fonctions trigonométriques réciproques



Images des fonctions trigonométriques réciproques

$y = \arcsin(x)$ $y \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$		$y = \arccos(x)$ $y \in [0, \pi]$	
$y = \arctan(x)$ $y \in \left]-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right[$		$y = \text{arccot}(x)$ $y \in]0, \pi[$	
$y = \text{arcsec}(x)$ $y \in [0, \pi] \backslash \left\{\dfrac{\pi}{2}\right\}$		$y = \text{arccsc}(x)$ $y \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right] \backslash \{0\}$	

Identités des fonctions trigonométriques réciproques

$$\sin^{-1}(\sin(x)) \equiv x \qquad \text{pour } -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$
$$\sin(\sin^{-1}(x)) \equiv x \qquad \text{pour } -1 \leq x \leq 1$$

$$\cos^{-1}(\cos(x)) \equiv x \qquad \text{pour } 0 \leq x \leq \pi$$
$$\cos(\cos^{-1}(x)) \equiv x \qquad \text{pour } -1 \leq x \leq 1$$

$$\tan^{-1}(\tan(x)) \equiv x \qquad \text{pour } -\frac{\pi}{2} < x < \frac{\pi}{2}$$
$$\tan(\tan^{-1}(x)) \equiv x \qquad \text{pour } -\infty < x < \infty$$

$$\cot^{-1}(x) \equiv \begin{cases} \tan^{-1}\left(\frac{1}{x}\right), & x > 0; \\ \pi + \tan^{-1}\left(\frac{1}{x}\right), & x < 0. \end{cases}$$

$$\sec^{-1}(x) \equiv \cos^{-1}\left(\frac{1}{x}\right) \qquad \text{pour } x \leq -1 \text{ ou } x \geq 1$$

$$\csc^{-1}(x) \equiv \sin^{-1}\left(\frac{1}{x}\right) \qquad \text{pour } x \leq -1 \text{ ou } x \geq 1$$